Linear-time Message-Passing Decoding of Quantum LDPC Codes

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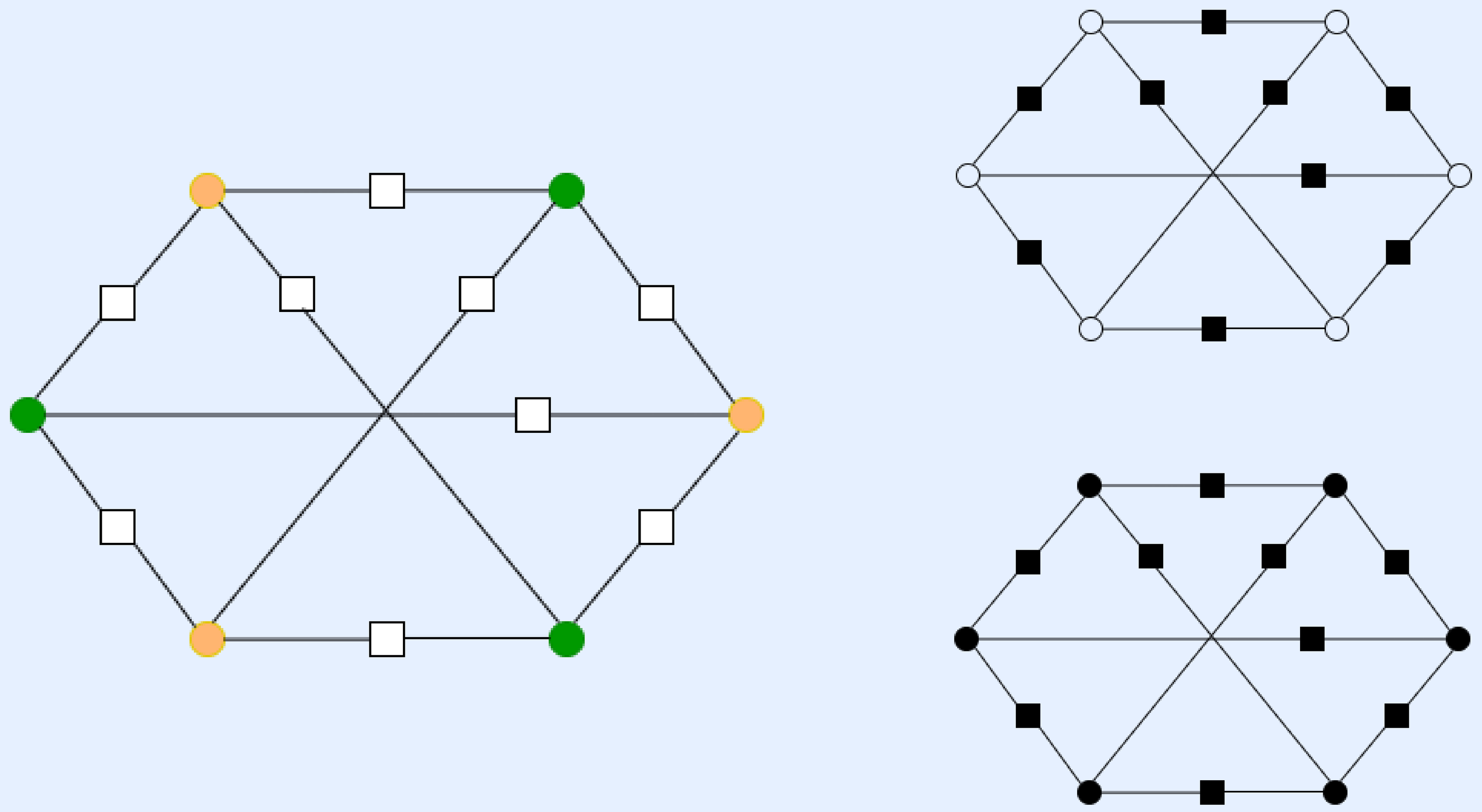
Background and Motivation

- **Quantum Error Correction (QEC)** aims to achieve high performance under strict latency constraints due to decoherence.
- **QLDPC** codes offer better minimum distance and code rate scaling than topological codes. At the cost of **increased qubit connectivity.**
- State-of-the-art QLDPC decoding relies on **serial scheduling or post-processing [1-3]**, which increase decoding latency and complexity.
- Iterative decoding is limited in QLDPC due to **short cycles and degeneracy**.
- Decoding failures can be characterized by **trapping sets** [4,5].
- Min-Sum (MS) decoding is a low-complexity alternative to Belief Propagation (BP).
  - We analyze the evolution of Min-Sum messages for the **lowest-weight degenerate errors**, where half of the variable nodes in a symmetric stabilizer are in error.
  - We propose a parallel Min-Sum decoder that exploits this degeneracy to achieve **linear-time decoding**.
  - Our analysis focuses on Bivariate Bicycle (BB) codes [6].



Decoder tries to estimate both error patterns that share the same syndrome. Output oscillates between including all 6 variable nodes or none at all.

[1] P. Panteleev and G. Kalachev, "Degenerate quantum LDPC codes with good finite length performance," Quantum, vol. 5, p. 585, Nov. 2021.

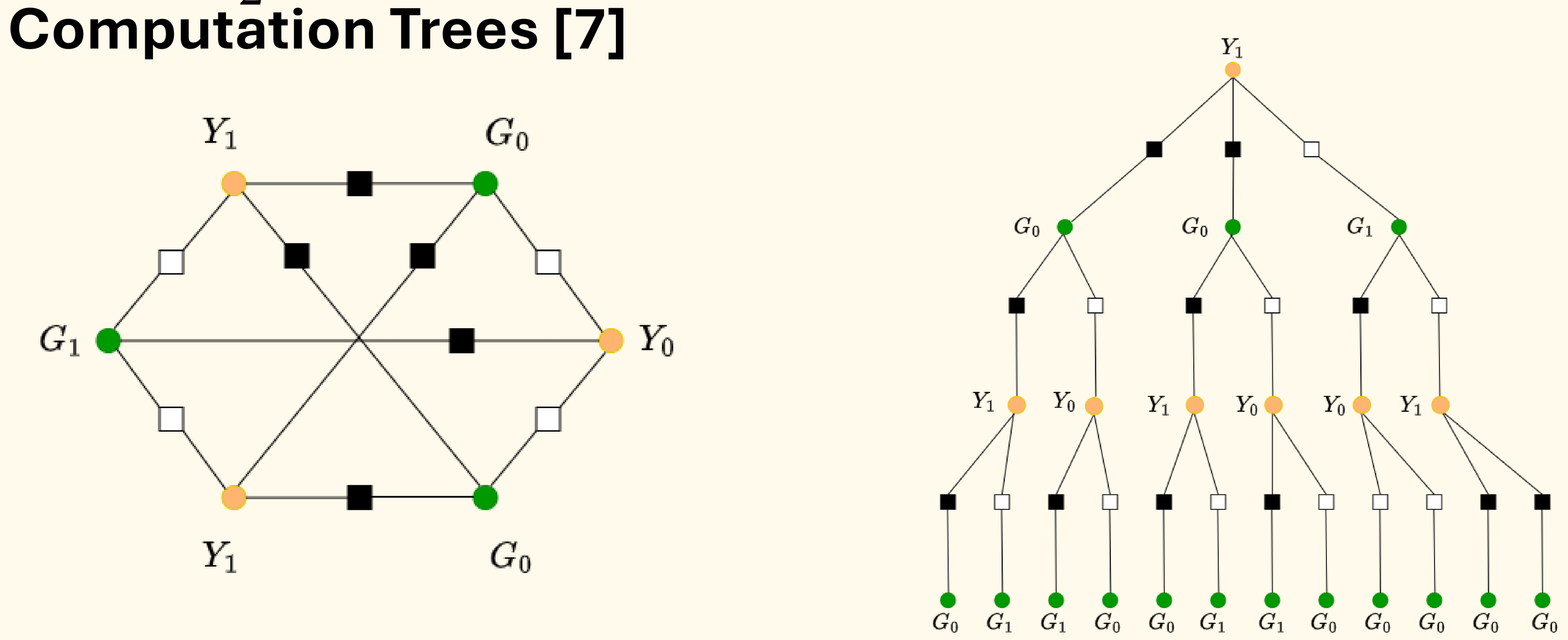
[2] J. Du Crest, F. Garcia-Herrero, M. Mhalla, V. Savin, and J. Valls, "Layered Decoding of Quantum LDPC Codes," in 2023 12th International Symposium on Topics in Coding (ISTC), 2023, pp. 1–5.

[3] H. Yao, W. A. Laban, C. H¨ager, A. G. i Amat, and H. D. Pfister, "Belief Propagation Decoding of Quantum LDPC Codes with Guided Decimation," arXiv preprint arXiv:2308.13377, 2023.

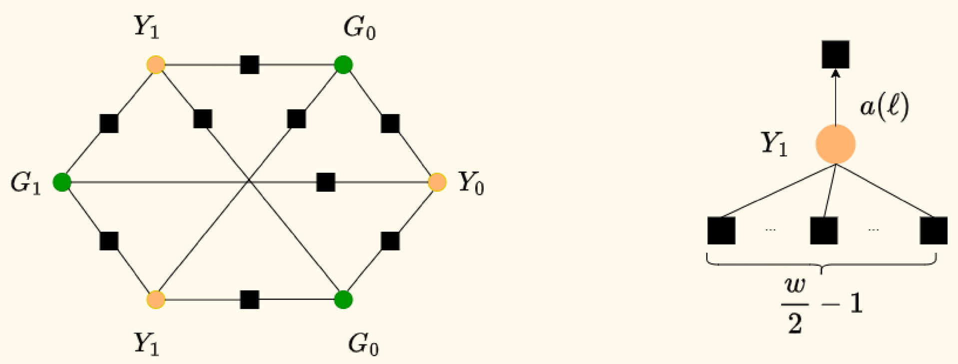
[4] Vasic, B. et al., "Trapping set ontology," Allerton Conf., 2009.

Min-Sum Analysis

- Tracking evolution of variable-to-check messages for every weight- $\frac{w}{2}$ error pattern inside $(w, 0)$ trapping sets : **Computation Trees [7]**



- Under **Isolation assumption**, MS reduces to a **discrete linear dynamical system.**
- **Case 1: Errors on variable nodes of one color**

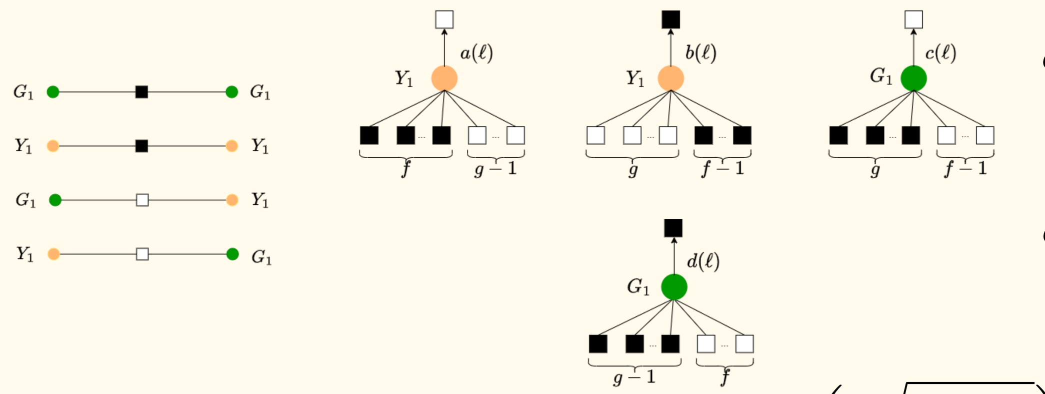


$$a(\ell) = -(\frac{w}{2} - 1) \cdot a(\ell - 1) + \lambda,$$

$$a(\ell) = -(\frac{w}{2} - 1) \cdot a(\ell - 1),$$

$$a(\ell) = \lambda \cdot (-\frac{w}{2} + 1)^{\ell}.$$

- **Case 2: Errors on variable nodes of both colors**



$$a(\ell) = (g - 1) \cdot c(\ell - 1) - f \cdot b(\ell - 1) + \lambda,$$
$$b(\ell) = g \cdot c(\ell - 1) - (f - 1) \cdot b(\ell - 1) + \lambda,$$
$$c(\ell) = (f - 1) \cdot a(\ell - 1) - g \cdot d(\ell - 1) + \lambda,$$
$$d(\ell) = f \cdot a(\ell - 1) - (g - 1) \cdot d(\ell - 1) + \lambda.$$

$$D_1 \mathbf{v}_1 \cdot 1^{\ell} + D_2 \mathbf{v}_2 \cdot \left(i\sqrt{\frac{w}{2} - 1}\right)^{\ell} + D_3 \mathbf{v}_3 \cdot \left(-i\sqrt{\frac{w}{2} - 1}\right)^{\ell} + D_4 \mathbf{v}_4 \cdot \left(1 - \frac{w}{2}\right)^{\ell},$$

- **Main takeaway:** Variable-to-check messages **change sign in every iteration**, with **amplitude growing** with respect to the weight of the stabilizer.
  - **Decoding insight**: Tracking oscillating messages can *"reveal" degenerate errors*.
  - **Anisotropy**: We *only track one block's messages, allowing the other to update normally.*

[5] Raveendran, N. and Vasic, B., "Trapping sets of quantum LDPC codes," Quantum 5, 562 (2021).

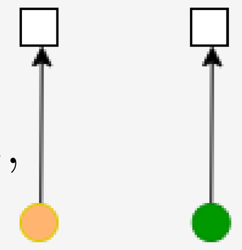
[6] Bravyi, S. et al., "High-threshold and low-overhead fault-tolerant quantum memory," Nature, 627.8005 (2024): 778–782.

[7] N. Wiberg, "Codes and decoding on general graphs," Ph.D. dissertation, Univ. Link¨oping, Sweden, Dept. Elec. Eng., 1996.

Min-Sum with Past Influence

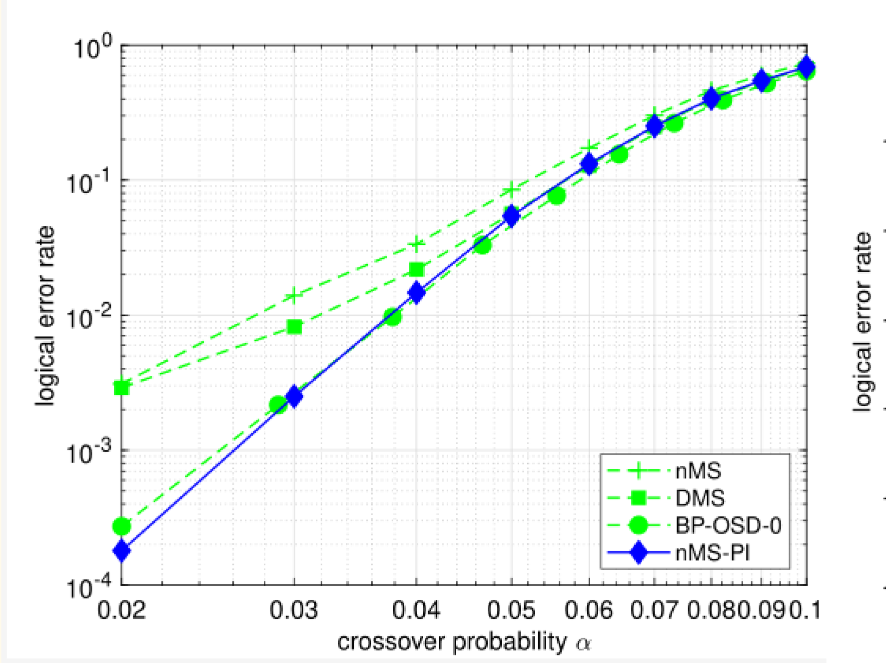
- Messages sent by "yellow" variable nodes are conventionally updated.
- Messages from green nodes apply **a damping mechanism, conditioned on the previous iteration**, to suppress **oscillations**.

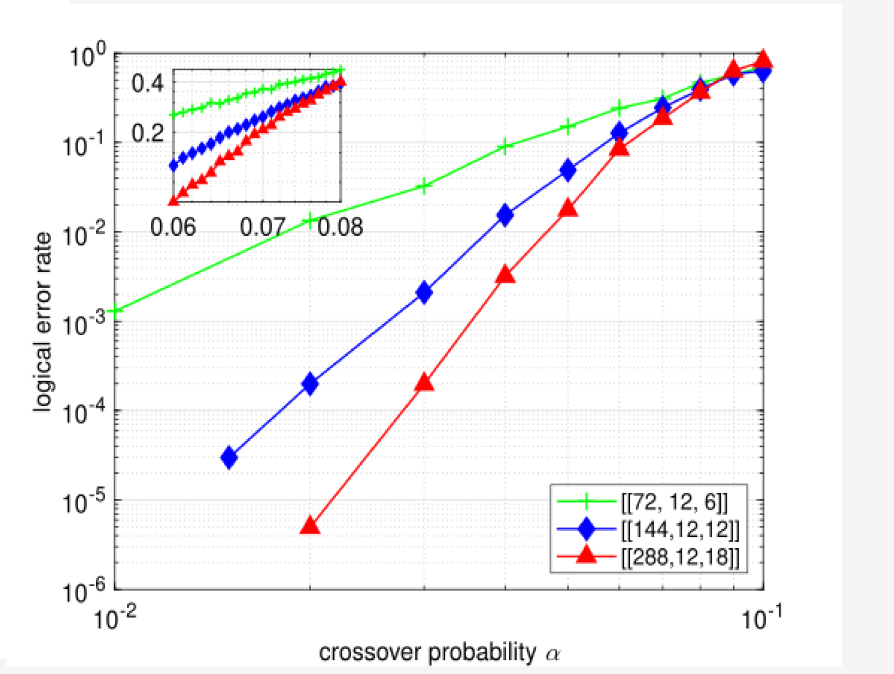
$$\nu_{j,i}^{(\ell)} = \lambda_j + \sum_{i' \in \mathcal{M}(j)\setminus\{i\}} \mu_{i',j}^{(\ell)},$$



$$\nu = \lambda_j + \sum_{i' \in \mathcal{M}(j)\setminus\{i\}} \mu_{i',j}^{(\ell)},$$

$$\nu_{j,i}^{(\ell)} = \begin{cases} \nu, & \text{if } (\mathrm{sgn}(\nu) = \mathrm{sgn}(\nu_{j,i}^{(\ell-1)})) \\ \nu + \nu_{j,i}^{(\ell-1)}, & \text{otherwise.} \end{cases}$$





- *Performance Evaluation*
  - Noise model: Binary Symmetric Channel (BSC) inducing X (bit-flip) errors.
  - MS-PI **outperforms BP-OSD-0** for the [[144,12,12]] BB code in only 50 iterations.
  - MS-PI achieves **a threshold of ~7.8%** for BB codes.
- *Conclusions/Impact*
  - Linear-complexity parallel-scheduled decoder breaks degeneracy.
  - For state-of-the art BB codes, MS-PI **outperforms** BP-OSD-0.
- *Future considerations*
  - Evaluation over more realistic channels:
    - Phenomenological noise model.
    - **Circuit-level noise-model**.
  - Speed-up decoding for near-distance trapping sets.

This work is available as a preprint at: **https://arxiv.org/abs/2501.05021**

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