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All-Photonic Quantum Repeaters with 9 km spacing

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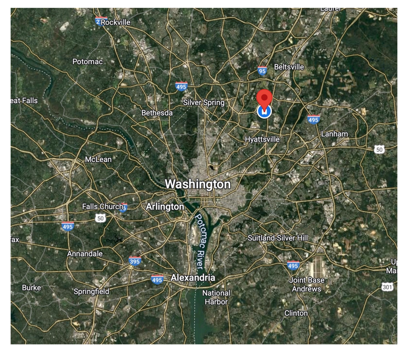
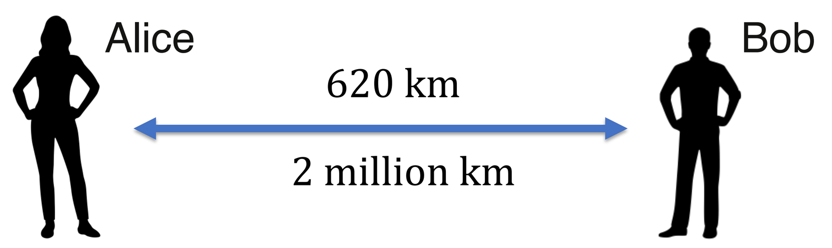
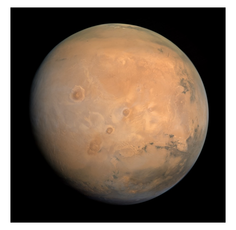
Motivation

Alice has a very important quantum bit, and wants to deliver it to Bob!





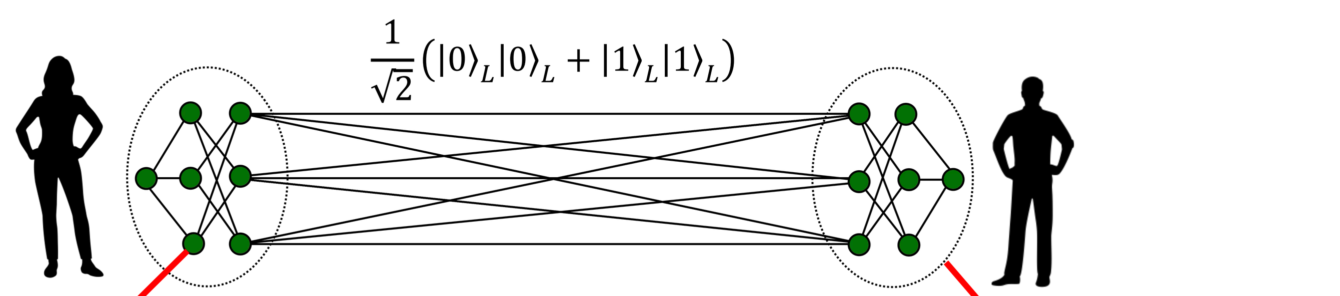


The most famous method: Quantum Teleportation

When they want to share identical secret classical bit strings; Quantum Key Distribution (QKD)

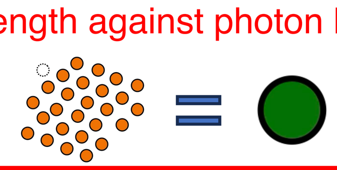
All quantum communication protocols requires
Pre-shared Entanglement Pairs Between them

GKP Code + [[7, 1, 3]] Steane Code



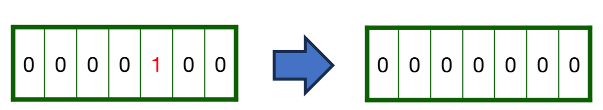
GKP Code

- 10^2 - 10^6 photons combine and represent one GKP qubit.
- GKP qubits are famous for its strength against photon loss!

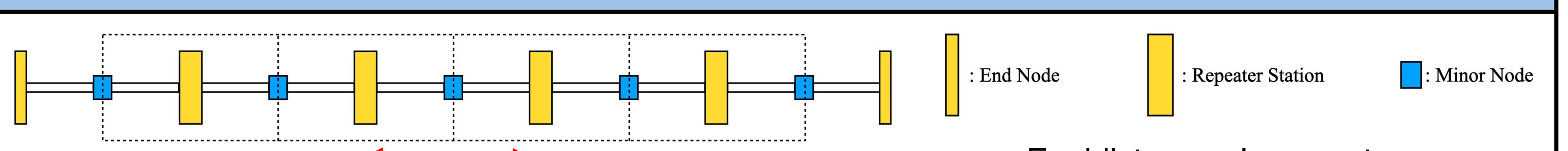


[[7, 1, 3]] Steane Code

- 7 GKP qubits combine and represent one [[7, 1, 3]] logical qubit.
- [[7, 1, 3]] Steane code alone can correct up to one bit-flip error!



Architecture



Equidistance placement

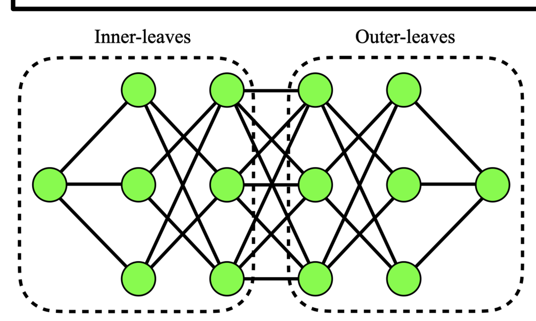
When $L_{\text{total}} = 1000km$, $L_0 = 9\ km$

2-9 times bigger than previous research

Much smaller optical modes and resources

c.f. Comparing to other famous schemes

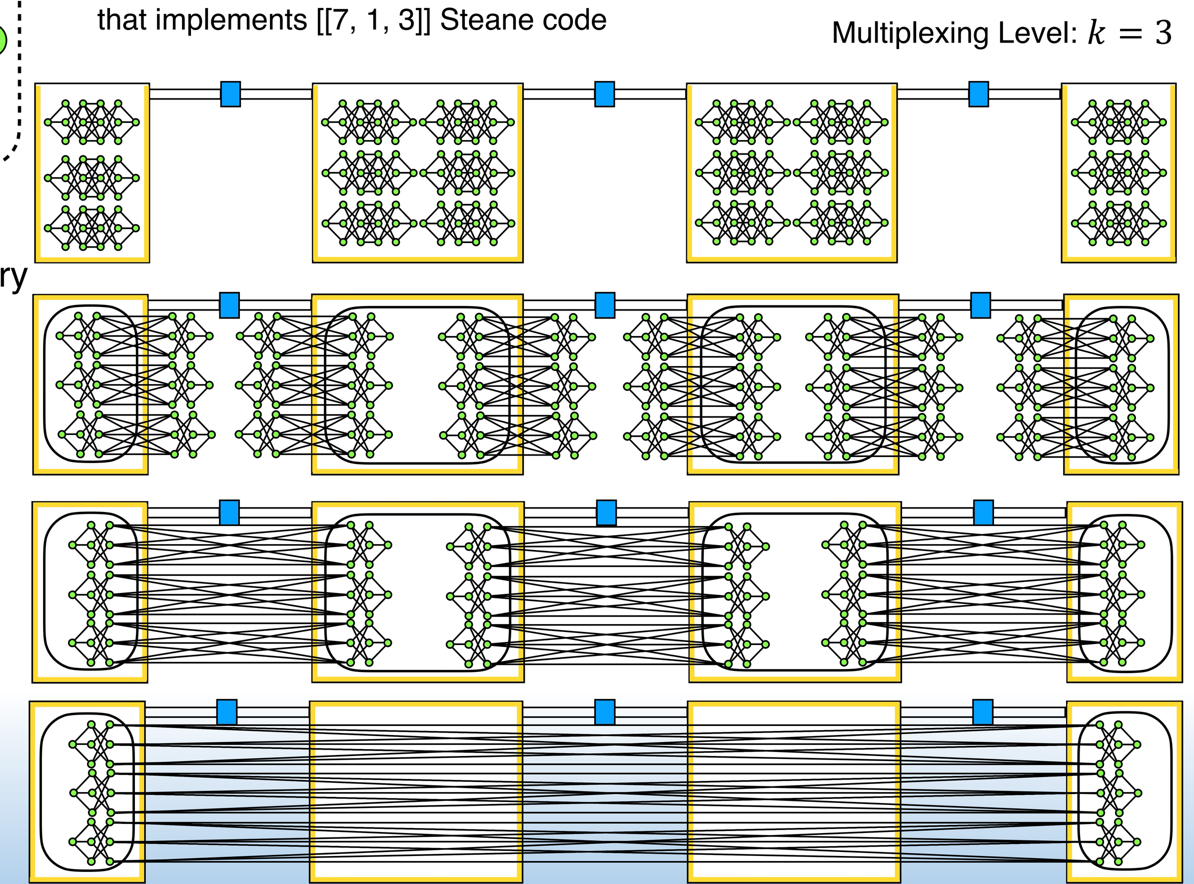
Azuma et al.; $L_0 = 4-8\ km$	Nat Commun 6, 6787 (2015).
Pant et al.; $L_0 = 1.5\ km$	Phys. Rev. A 95, 012304 (2017).
Fukui et al.; $L_0 = \sim 0.5 - \sim 1\ km$	Phys. Rev. Research 3, 033118 (2021).
Rozpędek et al.; $L_0 = 0.5-2.5\ km$	Rev. Research 5, 043056 (2023).



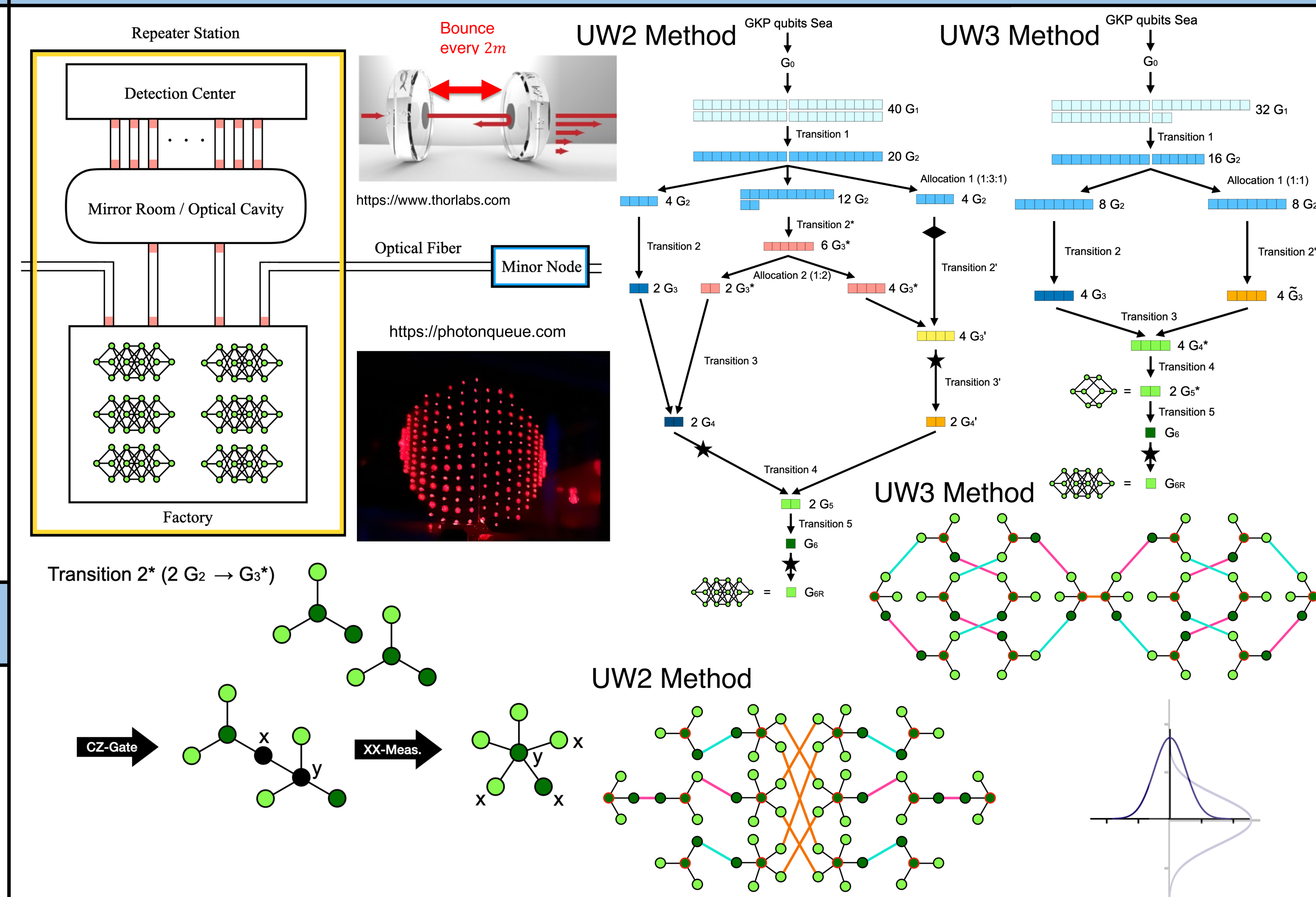
Minimum-Edges Graph States that implements [[7, 1, 3]] Steane code

Multiplexing Level: $k = 3$

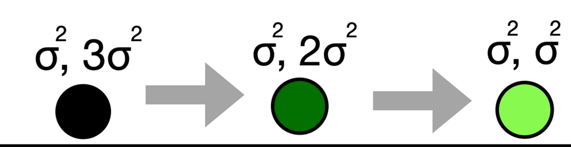
1. Construction of elementary entanglement pairs
2. Outer-leaves Swapping
3. Inner-leaves Swapping



Construction of Entangled Bell Pairs



In our scheme, we utilize 3 types of GKP qubits.



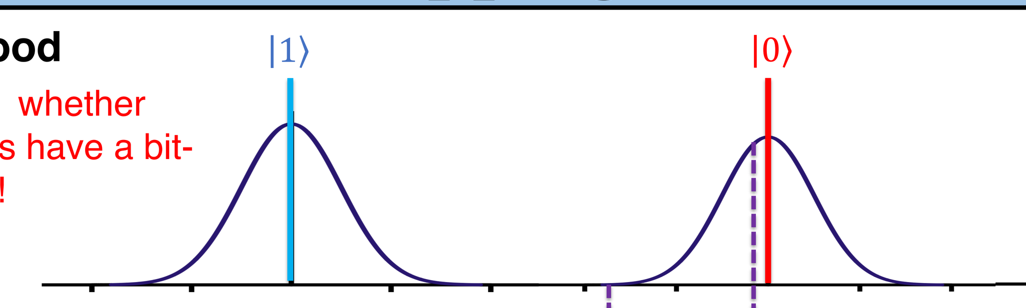
Inner-Leaves Swapping

GKP Outcome → Error Likelihood

$q_0 \to 0$ or 1
$p_0 \to 0$ or 1

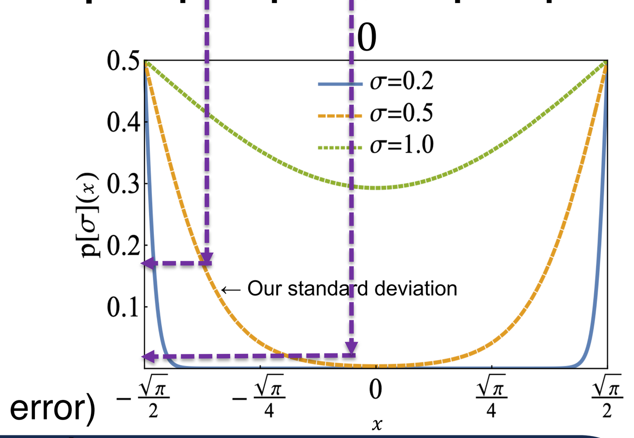
We don't know whether these outcomes have a bit-flip error or not!

However, we know its error likelihood!



$$p[\sigma](x) = \frac{\sum_{n\in\mathbb{Z}} \exp[-(x-(2n+1)\sqrt{\pi})^2/(2\sigma^2)]}{\sum_{n\in\mathbb{Z}} \exp[-(x-n\sqrt{\pi})^2/(2\sigma^2)]}$$

$$Z_{p_0} \equiv p[\sigma](p_0) \qquad Z_{q_0} \equiv p[\sigma](q_0)$$



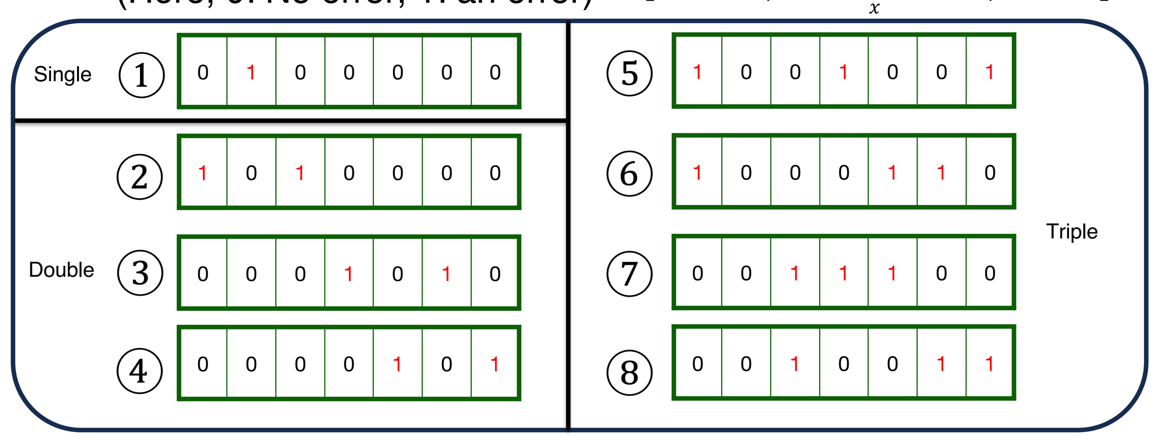
[[7, 1, 3]] syndrome → Error Pattern

We calculate [[7, 1, 3]] syndrome, which is a set of parities $IIIPPPP$, $IPPIIPP$, and $PIPIPIP$.

(Here, 0: No error, 1: an error)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
0	0	0	1	1	1	1
0	1	1	0	0	1	1
1	0	1	0	1	0	1

For example, $(0, 1, 0)$



Error Likelihood + Error Pattern = Total Error Likelihood

For each error pattern,

$$① \ 0\,1\,0\,0\,0\,0\,0 \quad \xi_1 = (1-Z_{q_{0,1}})Z_{q_{0,2}}(1-Z_{q_{0,3}})(1-Z_{q_{0,4}})(1-Z_{q_{0,4}})(1-Z_{q_{0,4}})$$

Should bit-flip / Shouldn't bit-flip

We estimate which error pattern actually occurred based on the following equation.

$$\xi_i(q) = \max_{\text{All patterns}} \{\prod_m \prod_n z^i_{q_0,m}(1-z^i_{q_0})_n\}$$

And correct this error pattern!

GKP Error Correction Code + [[7, 1, 3]] Steane Error Correction Code
↪ Tells us Error Likelihood ↪ Tells us Error Pattern

We can correct up to 3 bit-flip errors!

Outer-Leaves Swapping

$$\xi_i(q) = \max_{\text{All patterns}} \{\prod_m \prod_n z^i_{q_0,m}(1-z^i_{q_0})_n\}$$

$$\Xi_i(q) = \sum_{\text{All patterns}} \prod_m \prod_n z^i_{q_0,m}(1-z^i_{q_0})_n$$

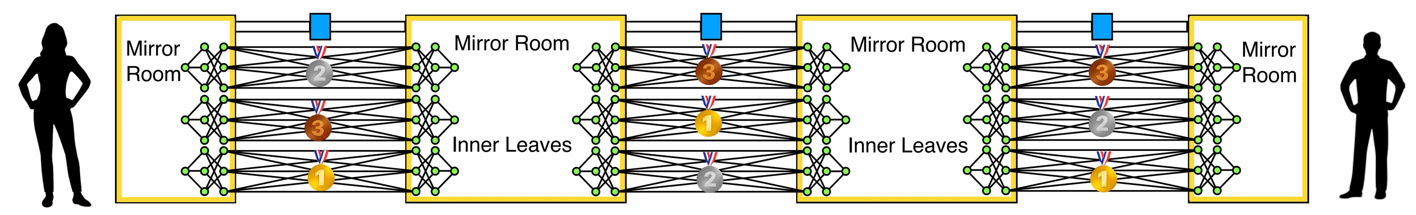
$$\Rightarrow r_i(q) = \frac{\xi_i(q)}{\Xi_i(q)}$$

We define the quality of each link as $Q_i = r_i(q) \cdot r_i(p)$.

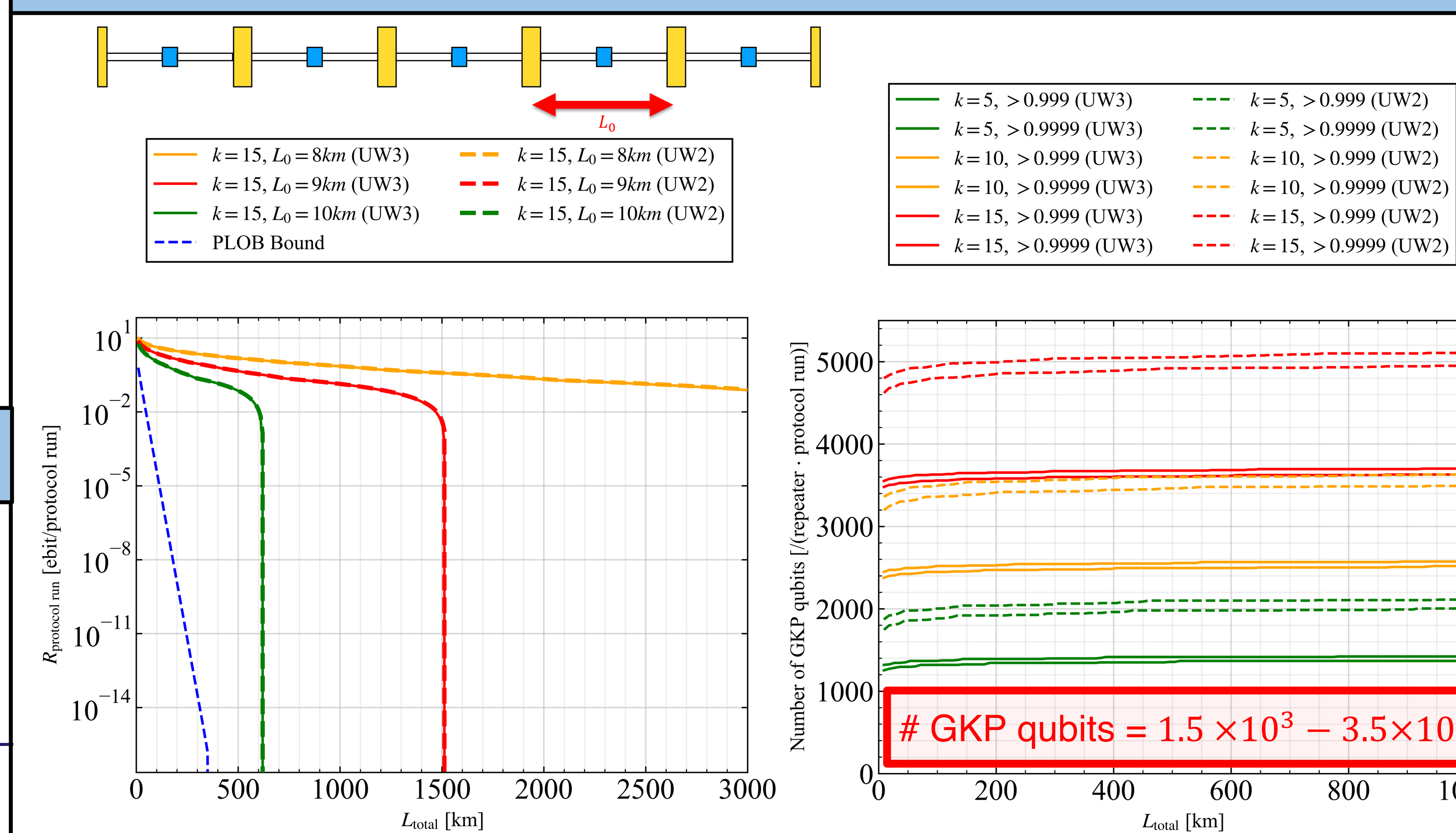
Based on these values, we rank our links

$$① = \max_{i\in\{1,\cdots,k\}}\{Q_i\} \quad ② = \max_{i\in\{1,\cdots,k\}}{}^{(2)}\{Q_i\} \quad ③ = \max_{i\in\{1,\cdots,k\}}{}^{(3)}\{Q_i\}$$

We connect the same rank links.

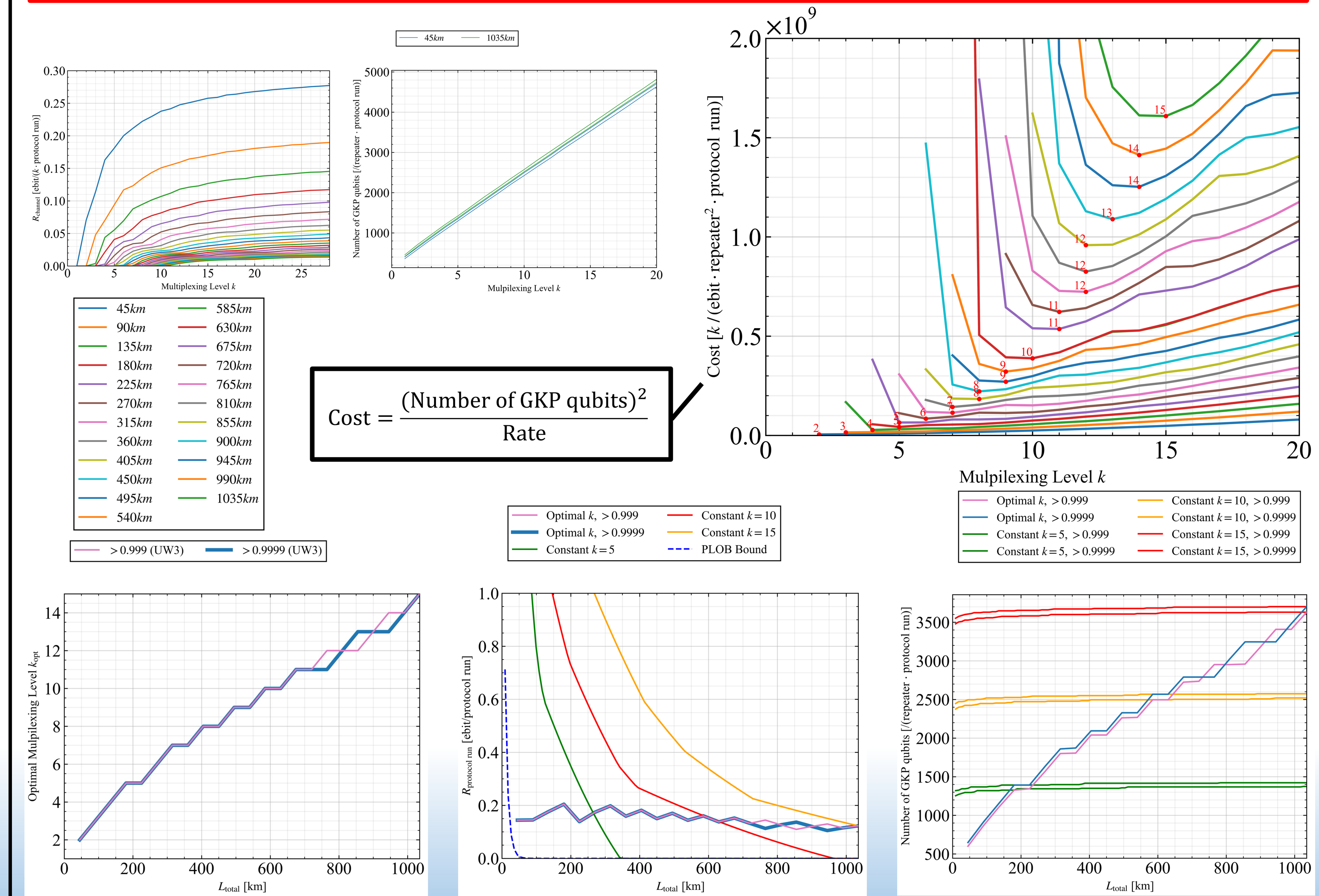


Results



c.f. When $L_{Total} = 1000km$ (per repeater)

$L_0 = 4-8\ km$,	# Single photons = $\sim 10^{24}$
$L_0 = 1.5\ km$,	# Single photons = 3.3×10^6
$L_0 = \sim 0.5 - \sim 1\ km$	# GKP qubits = $> 4.3 \times 10^4$ (Without construction process)
$L_0 = 0.5 - 2.5\ km$,	# GKP qubits = 5×10^3 ($L_0 = 0.5km$)



$$\text{Cost} = \frac{(\text{Number of GKP qubits})^2}{\text{Rate}}$$