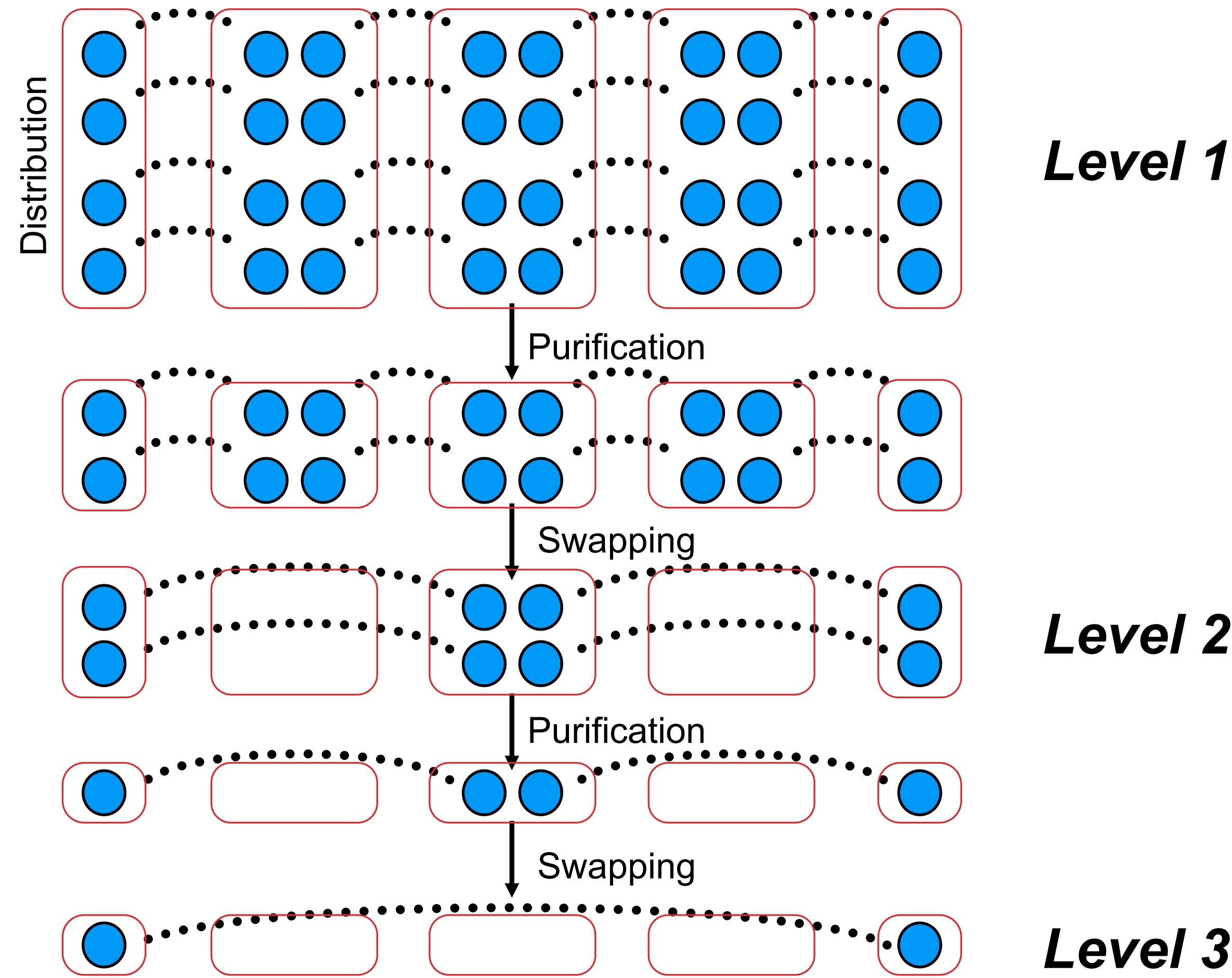




Introduction and Motivation

The goal of the project is to find error correction-based distillation protocols that optimize performance metrics, such as end-to-end entanglement rate and fidelity.



Independent of the number of repeaters, we always perform only up to three rounds of distillation.

Simulation Setup and Assumptions

Step 1: Link-Level Entanglement Generation

EPR pairs (Bell pairs) generated at each link

Step 2: Depolarizing Channel

Bell pairs generated at each link pass through a depolarizing channel, introducing noise that converts them into depolarized states

Step 3: Quantum Memory

After entanglement generation, photons are stored in quantum memories at repeaters and nodes, preserving entanglement for subsequent operations

Step 4: Entanglement Distillation

Once stored in quantum memories, CNOT gates and measurements distill entanglement, using stabilizer codes to enhance output fidelity

Step 5: Entanglement Swapping

After distillation, entanglement swapping connects links to extend entanglement distance; additional swaps or distillations may follow per policy

Step 6: Final Entangled States

After up to three rounds of distillation and swapping, final entangled states form at end nodes for quantum communication or computation

Simulation Framework

Noise

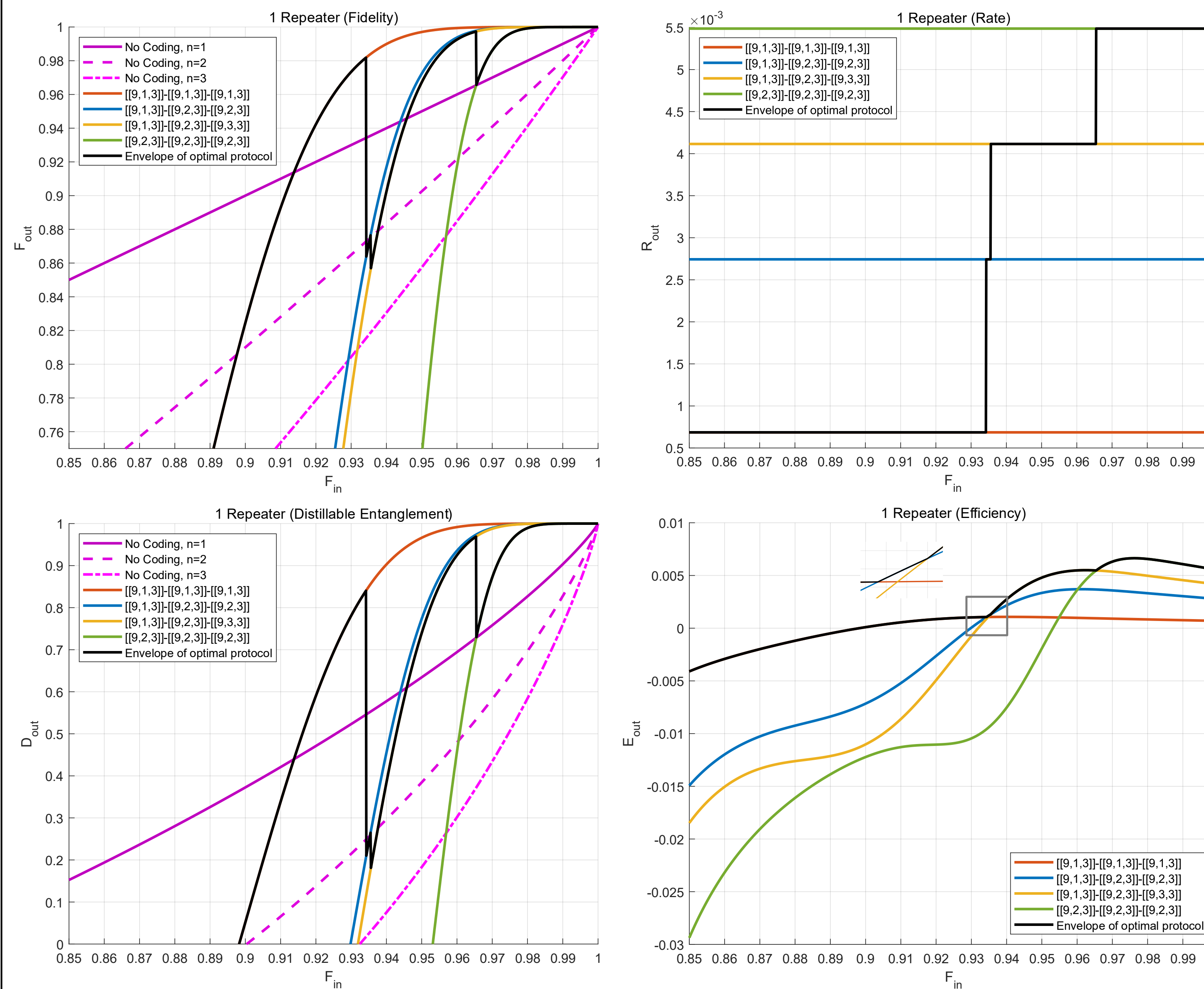
Distillation Policy

Codes

Capabilities of the software framework:

- Configure different noise for each link
- Apply different policies of distillation
- Set different codes for each link and each round

2G Distillation (QEC) with Adaptive Strategy



• Fidelity:

• The protocol rate:

$$R_{out} = \frac{n_{out}}{n_{in}}$$

n_{in} – The number of entangled states consumed

n_{out} – The number of output entangled states

• The distillable entanglement:

$$D(F) = 1 + F \log_2 F + (1 - F) \log_2 \left(\frac{1 - F}{3} \right)$$

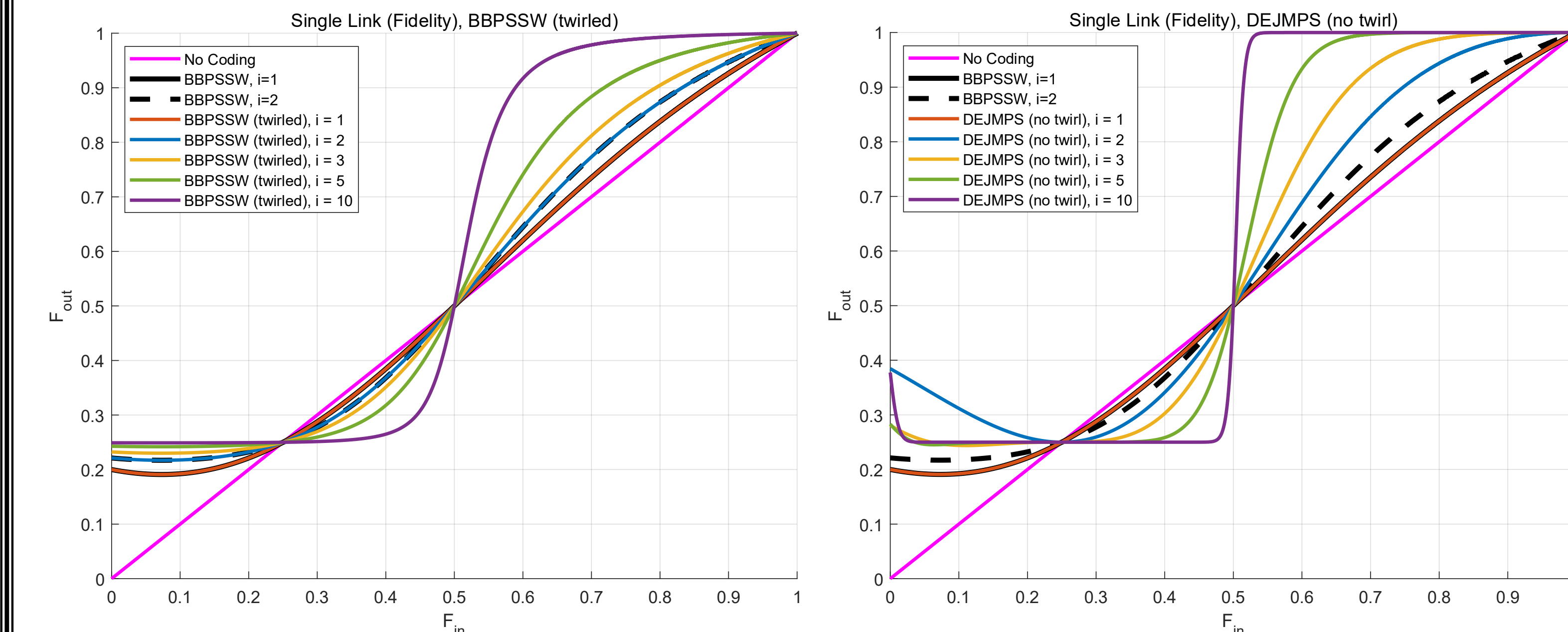
• The efficiency we defined:

$$E(F_{in}) = \frac{n_{out} D(F_{out})}{n_{in} D(F_{in})}$$

PATENT APPLICATION

Systems and Methods for Adaptive Error Correction for Entanglement Distillation in Quantum Networks, Provisional Patent Application No. 63/788,694, filed April 14, 2025, University of Arizona.

1G Purification combined with 2G Distillation (QEC)



BBPSSW (twirled):

$$F_{out} = \frac{F_{in}^2 + \frac{1}{9}(1 - F_{in})^2}{F_{in}^2 + \frac{2}{3}F_{in}(1 - F_{in}) + \frac{5}{9}(1 - F_{in})^2}$$

The black line is based on the formula, while the others are obtained from our own circuit simulator

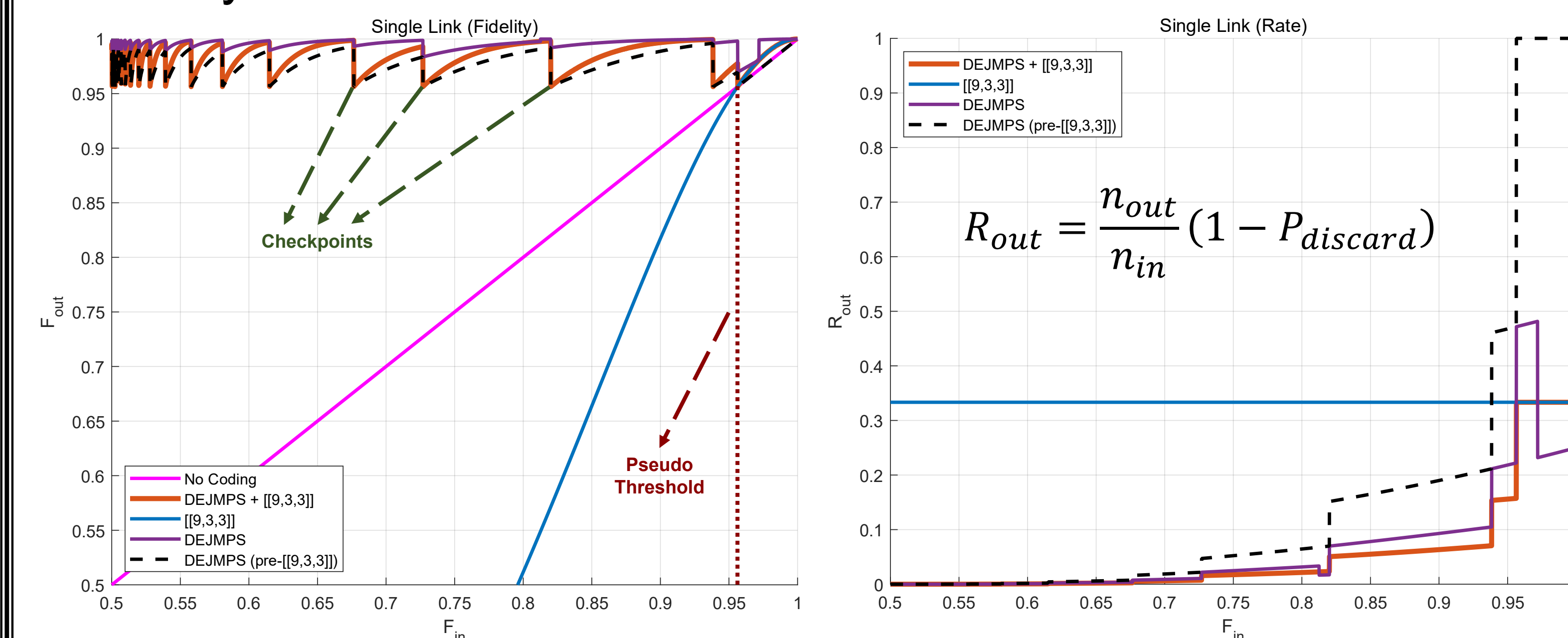
DEJMPS (no twirl):

$$P_I = P_{I_{in}}^2 + P_{Y_{in}}^2 \quad P_X = P_{X_{in}}^2 + P_{Z_{in}}^2 \quad P_Y = 2P_{X_{in}}P_{Z_{in}} \quad P_Z = 2P_{I_{in}}P_{Y_{in}}$$

$$P_{discard} = 1 - P_I - P_X - P_Y - P_Z$$

$$F_{out} = P_{I_{out}} = \frac{P_I}{1 - P_{discard}}; P_{A_{out}} = \frac{P_A}{1 - P_{discard}}, A = X, Y, Z$$

Starting from the second round, DEJMPS achieves higher fidelity than BBPSSW for the same number of rounds



- Checkpoints: Number of DEJMPS rounds decreases by 1 or 2
- DEJMPS+[[9,3,3]]: 1 QEC round after DEJMPS rounds

DEJMPS has higher fidelity than DEJMPS+[[9,3,3]] most times

Ongoing Work

Construct optimal sequences of codes with DEJMPS to optimize end-to-end performance metrics under realistic noise

REFERENCES

- C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, "Mixed-state entanglement and quantum error correction," Phys. Rev. A, vol. 54, no. 5, pp. 3824–3851, Nov. 1996. [Online]. Available: <https://doi.org/10.1103/PhysRevA.54.3824>
- N. Rengaswamy, N. Raveendran, A. Raina, and B. Vasić, "Entanglement Purification with Quantum LDPC Codes and Iterative Decoding," Quantum, vol. 8, p. 1233, Jan. 2024. [Online]. Available: <https://doi.org/10.22331/q-2024-01-24-1233>
- Sijie Cheng and Narayanan Rengaswamy, "Adaptive Error Correction for Entanglement Distillation," arXiv preprint arXiv:2504.11670, April 2025. [Online]. Available: <https://arxiv.org/abs/2504.11670>